

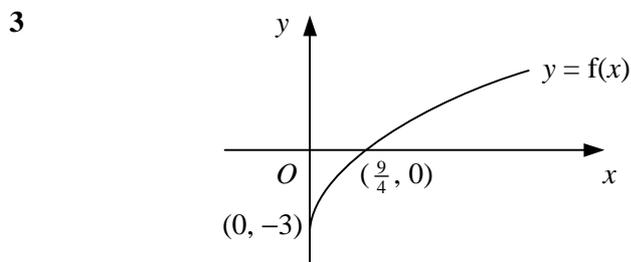
FUNCTIONS

- 1 $f: x \rightarrow 2 + \log_4 x, x \in \mathbb{R}, x > 0.$
- a Evaluate $ff(1).$ (3)
- b Solve the equation $f(x) = 0.$ (2)
- c Find the inverse function $f^{-1}(x).$ (3)

- 2 The function f is defined by
- $$f: x \rightarrow |3x - a|, x \in \mathbb{R}.$$

where a is a positive constant.

- a Find $ff(-2a).$ (2)
- b Sketch the graph $y = f(x)$, showing the coordinates of any points where the graph meets the coordinate axes. (3)
- c Solve the equation $f(x) = x$, giving your answers in terms of $a.$ (3)



The diagram shows the graph of $y = f(x)$ which meets the x -axis at the point $(\frac{9}{4}, 0)$ and the y -axis at the point $(0, -3).$

- a Sketch on separate diagrams the graphs of
- i $y = |f(x)|,$
- ii $y = f^{-1}(x).$ (4)
- Given that $f(x)$ is of the form $f(x) \equiv ax^{\frac{1}{2}} + b, x \in \mathbb{R}, x \geq 0,$
- b find the values of the constants a and $b,$ (3)
- c find an expression for $f^{-1}(x).$ (3)

- 4 The function f is defined by
- $$f: x \rightarrow \frac{x+2}{x-1}, x \in \mathbb{R}, x \neq 1.$$
- a Show that $ff(x) = x$ for all $x \in \mathbb{R}, x \neq 1.$ (3)
- b Hence, write down an expression for $f^{-1}(x).$ (1)

The function g is defined by

$$g: x \rightarrow 2x - 3, x \in \mathbb{R}.$$

- c Solve the equation $gf(x) = 0.$ (4)
- 5 a Sketch on the same set of axes the graphs of $y = |x|$ and $y = |2x - 3|.$ (3)
- b Hence, or otherwise, solve the equation
- $$|x| = |2x - 3|. \quad (4)$$

- 6 The function $f(x)$ is defined for all real values of x by

$$f(x) = x + 2, \quad x < 1,$$

$$f(x) = 4 - x^2, \quad x \geq 1.$$

- a Sketch the graph of $f(x)$ showing the coordinates of any points of intersection with the coordinate axes. (4)
- b Evaluate $ff(3)$. (2)
- c Solve the equation $f(x) = 1$. (4)

- 7 The functions f and g are defined by

$$f : x \rightarrow kx + 2, \quad x \in \mathbb{R},$$

$$g : x \rightarrow x - 3k, \quad x \in \mathbb{R},$$

where k is a constant.

- a Find expressions in terms of k for

i $f^{-1}(x)$,

ii $fg(x)$. (4)

Given that $fg(7) = 4$,

- b find the two possible values of k . (3)

- 8 $f(x) \equiv x^2 - 4x + 5, \quad x \in \mathbb{R}, \quad x \geq 2.$

- a Express $f(x)$ in the form $a(x + b)^2 + c$. (2)
- b State the range of f . (1)
- c Find an expression for $f^{-1}(x)$ and state its domain. (4)
- d Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between the graphs. (4)

- 9 The functions f and g are defined by

$$f : x \rightarrow x^2 + 4, \quad x \in \mathbb{R},$$

$$g : x \rightarrow 2x - \frac{1}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

- a Evaluate $gf(-2)$. (2)
- b Find and simplify an expression for $fg(x)$. (3)
- c Find the values of x for which $fg(x) = 5$. (4)

- 10 The function f is given by

$$f : x \rightarrow e^{\frac{1}{2}x} - 3, \quad x \in \mathbb{R}.$$

- a Find $f^{-1}(x)$ and state its domain. (4)
- b Sketch the curve $y = f^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes. (3)

The function g is given by

$$g : x \rightarrow \ln(x + 5), \quad x \in \mathbb{R}, \quad x > -5.$$

- c Evaluate $fg(4)$. (2)
- d Solve the equation $f^{-1}(x) = g(x)$. (4)